How Do You Solve a Quadratic Equation With Complex Solutions by Using the Quadratic Formula?

Use the Quadratic Formula to solve $x^2 - 4x + 5 = 0$ for x.

www.VirtualNerd.com

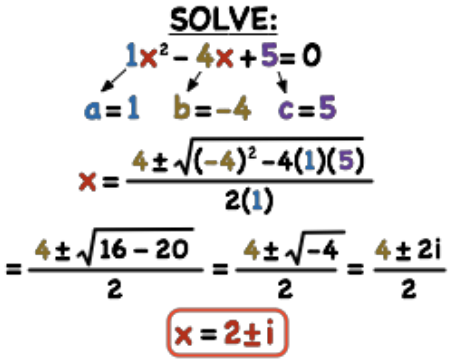
SUMMARY

Since there is no coefficient in front of x^2, that means the 'a' value is 1

When we plug in for 'b' we're taking the negative of -4, which gives us a positive 4

'±' means 'plus or minus', which means we could end up with two answers depending on if we add or subtract

'i' is the imaginary unit, and it is equal to the square root of -1



NOTES

Step 1) Make sure the equation is set equal to 0

The Quadratic Formula only works if the equation is set equal to 0!

That means we need to have 0 on one side and all the other terms on the other side

Step 2) Identify a, b, and c

'a' is the coefficient in front of the x^2 term

Since there is no coefficient in front of x^2, that means the 'a' value is 1

'b' is the coefficient in front of the x term, which is -4

Remember, the sign of the coefficient needs to stay with it!

'c' is the constant term, which is 5

The general form for a quadratic equation is:

$$ax^2+bx+c=0$$

'a' is the coefficient in front of the x^2 term

Since there is no coefficient in front of x^2, that means the 'a' value is 1

'b' is the coefficient in front of the x term, which is -4

Remember, the sign of the coefficient needs to stay with it!

'c' is the constant term, which is 5

Step 3) Plug into the Quadratic Formula

Now that we know what a, b, and c are, we can plug those values into the Quadratic Formula!

The Quadratic Formula is:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

The symbol above the 'b^2-4ac' is the symbol for square root

'±' means 'plus or minus'

Now that we know what a, b, and c are, we can plug those values into the Quadratic Formula!

The a, b, and c values in the Quadratic Formula are the same as the ones we found in our quadratic equation

So we can plug 1 in for 'a', -4 in for 'b', and 5 in for 'c'

Step 4) Simplify

First simplify underneath the square root

$(-4)^2$ is 16, and -4•1•5 is -20

Subtracting 16-20 gives us a negative, -4, under the square root

When we simplify we get a negative number under the radical!

This means we will have two complex solutions for our answers!

'Radical' is another term for the square root symbol

Since we have the square root of a negative, we will have an 'i' in our answer

Remember, 'i' is the imaginary unit, and it is equal to the square root of -1

We can split up our radical into √(4)•√(-1)

The square root of 4 is 2, and the square root of -1 is 'i'

So √(-4) simplifies to 2i

Dividing the top and bottom by 2 gives us a simplifed answer of 2±i

This means our two solutions are 2+i and 2-i

Step 5) Check your answers

We need to plug 2+i and 2-i into the original equation to make sure we get true statements

If we do, then we know our answers are correct!

$x = 2+i$

$= (2+i)^2 - 4(2+i) + 5$

$= (2+i)(2+i) - 8 - 4i + 5$

$= 4 + 2i + 2i + i^2 - 8 - 4i + 5$

$= \cancel{4} + \cancel{2i} + \cancel{2i} + (-1) - \cancel{8} - \cancel{4i} + \cancel{5}$

$\left.\begin{array}{l} 4 + (-1) - 8 + 5 = 0 \\ 2i + 2i - 4i = 0 \end{array}\right\} \; 0 + 0 = 0$ ✓

Plug '2+i' in anywhere you see an 'x'

We can square complex numbers just like real numbers - just multiply 2+i times itself

Distribute the -4 into the parentheses to get -8-4i

FOILing together 2+i and 2+i gives us 4 for the First terms, 2i for the Outer terms, 2i for the Inner terms, and i^2 for the Last terms

i^2 is the same as -1

Terms with 'i' in them are like terms, just as if 'i' was a variable

Our original equation was equal to 0, so 2+i is a correct solution!

$x = 2 - i$

$= (2-i)^2 - 4(2-i) + 5$

$= (2-i)(2-i) - 8 + 4i + 5$

$= 4 - 2i - 2i + i^2 - 8 + 4i + 5$

$= 4 - 2i - 2i + (-1) - 8 + 4i + 5 = 0$ ✔

Plug '2-i' in anywhere you see an 'x'

We can square complex numbers just like real numbers - just multiply 2-i times itself

Distribute the -4 into the parentheses to get -8+4i

FOILing together 2-i and 2-i gives us 4 for the First terms, -2i for the Outer terms, -2i for the Inner terms, and i^2 for the Last terms

i^2 is the same as -1

Terms with 'i' in them are like terms, just as if 'i' was a variable

Our original equation was equal to 0, so 2-i is a correct solution!